Vehicle Road Keeping With No Driver Input

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1 Abstract

This paper introduces a vehicle controller that improves on the basic path following controller discussed in lecture. The basic controller can only follow an arbitrary function Y(X) in inertial coordinates but this does not allow for roads to curve more than 90° around. The main goal of this paper is to present the design and simulation results of a path following controller for any x-y parametric curve in time or, more generally, any arbitrary series of x-y coordinates in time. The controller also allows for skidding during tight turns at high speed while still tracking the given path, within the limits of the vehicle.

2 Introduction

Problem: A vehicle needs to autonomously follow a track known ahead of time and possible adjusted on the fly. Also needing the route to have consistent timing to reach certain points.

Why this is important: Autonomous vehicles are vehicles are about to become more prevalent and start doing things that previously required human drivers for. For example, a bus line that follows a fixed path every day and is required to arrive at every stop on time.

Similar controllers addressed in the past: In class, a path error controller able to follow a path defined as a function Y(X) in the inertial frame was defined. This is not flexible and limited to relatively straight roads or a short look-ahead distance.

Solution: Design a controller that compares its current state with where, what direction, and what speed it needs to be going and controls the steering and throttle to correct for errors. The controller is flexible, only requiring a series of X and Y coordinates. The desired speed, yaw, and other values are computed from just X and Y.

Structure: In this paper, I will first describe the dynamics of the system plant being controlled. This will be followed by an analysis of the problem and the control design to solve it. Finally I will show the results of the controller working successfully in simulation.

3 Plant Description

The plant model used for simulation is a 7-DOF vehicle model, a four-wheel bicycle model. This model takes into account body fixed velocity ($\dot{x}$ and $\dot{y}$), yaw rate ($\dot{\psi}$), and all four-wheel speeds (w_{fl}, w_{fr}, w_{rl}, and w_{rr}) as state variables. The state variable update equations are as follows derived from Rajamani pg. 225 and 228:

$$\ddot{x} = \frac{1}{m}\left[\left(F_{xfl} + F_{xfr}\right)cos(\delta) + F_{xrl} + F_{xrr} - \left(F_{yfl} + F_{yfr}\right)sin(\delta)\right] + \dot{\psi}\dot{y} \tag{1}$$

$$\ddot{y} = \frac{1}{m}\left[F_{yrl} + F_{yrr} + \left(F_{xfl} + F_{xfr}\right)sin(\delta) + \left(F_{yfl} + F_{yfr}\right)cos(\delta)\right] - \dot{\psi}\dot{x} \tag{2}$$

$$\ddot{\psi} = \frac{1}{I_z}\Big[l_f\left(F_{xfl} + F_{xfr}\right)sin(\delta) + l_f\left(F_{yfl} + F_{yfr}\right)cos(\delta) - l_r\left(F_{yrl} + F_{yrr}\right) + \frac{l_w}{2}\left(F_{xfr} - F_{xfl}\right)cos(\delta) + \frac{l_w}{2}\left(F_{xrr} - F_{xrl}\right) + \frac{l_w}{2}\left(F_{yfl} - F_{yfr}\right)\Big] \tag{3}$$

$$\dot{\omega}_{fl} = \frac{1}{J_w}\left[T_{dfl} - T_{bfl} - r_{eff}F_{xfl}\right] \tag{4}$$

$$\dot{\omega}_{fr} = \frac{1}{J_w}\left[T_{dfr} - T_{bfr} - r_{eff}F_{xfr}\right] \tag{5}$$

$$\dot{\omega}_{rl} = \frac{1}{J_w}\left[T_{drl} - T_{brl} - r_{eff}F_{xrl}\right] \tag{6}$$

$$\dot{\omega}_{rr} = \frac{1}{J_w}\left[T_{drr} - T_{brr} - r_{eff}F_{xrr}\right] \tag{7}$$

The tire forces, F_{xfl}, F_{xfr}, F_{xrl}, F_{xrr}, and are modeled using the Dugoff Tire Force Model (Rajamani pg. 227):

$$F_x = C_\sigma \frac{\sigma}{1+\sigma} f(\lambda) \tag{8}$$

$$F_y = C_\alpha \frac{\tan\alpha}{1+\sigma} f(\lambda) \tag{9}$$

$$\text{where } \lambda = \frac{\mu F_z(1+\sigma)}{2\sqrt{(C_\sigma\sigma)^2 + (C_\alpha \tan\alpha)^2}}, \tag{10}$$

$$f(\lambda) = \begin{cases}(2-\lambda)\lambda, if\ \lambda < 1 \\ 1, if\ \lambda \geq 1\end{cases}, \tag{11}$$

$$\text{and } F_z = \frac{1}{4}mg \tag{12}$$

Where α and σ are the slip angle and slip ratio of the wheel; C_α and C_σ are the cornering stiffness and longitudinal tire stiffness respectively.

Assumptions about the vehicle:

- Only the front steering angle, δ, can be controlled with both front wheels at the same angle. The rear wheels are fixed at an angle of zero.
- The vehicle is front wheel drive. Zero torque can be applied from the rear wheels and only equal torques can be applied from the front wheels: $T_{dfl} = T_{dfr}$ and $T_{drl} = T_{drr} = 0$.
- All brake torques are the same, controlled centrally: $T_{bfl} = T_{bfr} = T_{brl} = T_{brr}$.

- The desired wheel torque is delivered from the engine immediately with no time delay or lag. (A simplification for this plant model.)

The vehicle controller will control the throttle, $T = T_{dfl} = T_{dfr}$, and steering angle, δ, inputs to this plant.

4 Analysis and Control Design

In analyzing the problem and choosing a control strategy for this problem, I first define error metrics for how closely the vehicle is following the given path. The two metrics I choose were e_1 and e_2, longitudinal path error and yaw error respectively, defined by relation between the path and the *vehicle velocity vector*, not the vehicle body-fixed frame. This allows for the vehicle to skid or drift around a turn without requiring the axis of the vehicle to be aligned with the road.

Following are the formal definitions of the errors.

$$e_1 = [X_{des} - X, Y_{des} - Y] \cdot [\dot{x}\cos\psi - \dot{y}\sin\psi\,, \dot{x}\sin\psi + \dot{y}\cos\psi] \quad (13)$$

$$e_2 = \psi_{vel,des} - \psi_{vel} \quad (14)$$

$$\text{where } \psi_{vel,des} = atan2(V_{Y,des}, V_{X,des}), \quad (15)$$

$$\text{and } \psi_{vel} = atan2(\dot{x}\sin\psi + \dot{y}\cos\psi\,, \dot{x}\cos\psi - \dot{y}\sin\psi) \quad (16)$$

e_2 is limited to be between –π and +π through the modulus.

From these definitions, using proportional control, I derive desired vehicle longitudinal acceleration and steering angle.

$$\ddot{x}_{control} = k_{p0}\sqrt{{V_{X,des}}^2 + {V_{Y,des}}^2} + k_{p1}e_1 \quad (17)$$

$$\delta_{control} = \delta_{start} + k_{p2}e_2 \quad (18)$$

$$\text{where } \delta_{start} = \frac{1}{R}\left[l_f + l_r + (m\dot{x}^2(l_r - l_f)C_\alpha)/(2{C_\alpha}^2(l_r + l_f)\right] \quad (19)$$

δ_{start} is the predicted steering angle from the given road radius of curvature, R and the error term is used to correct any offset this ideal turning radius may have from reality.

The values of the three proportionality constants were tuned by hand and found to be 2, 0.04, and 1 respectively.

5 Simulation Result

The initial conditions are in SI units and are as follows:

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[x_dot, y_dot, psi,  psi_dot, w_fl,   w_fr,   w_rl,   w_rr,   X, Y];
[10,    0,     pi/2, 0,       w_init, w_init, w_init, w_init, 40, 0];
```

where w_init is calculated from the initial speed of 10m/s and the wheel radius.

The vehicle parameters are as follows:

```
P.l = 2.5;                    % Car length [m]
P.l_f = 0.4*P.l;              % CoG front axle distance [m]
P.l_r = P.l - P.l_f;          % CoG rear axle distance [m]
P.l_w = 1.539;
P.m = 1274;                   % vehicle sprung mass [Kg]
P.I_z = 1523;                 % yaw moment of inertia [kgm^2]
P.g = 9.81;                   % Gravity
P.mu = 0.9;                   % friction coefficient
P.C_alpha = 5500;             % Cornering Stiffness [N/rad]
P.C_sigma = 3000;             % Tire Stiffness []
P.r_eff = 0.305;              % Effective raduis of tire.
P.J_w = 1;                    % Rotational Inertia of Tire
P.rho = 0.25;                 % Front/Rear Break ratio.
P.eta = 15;                   % Sliding Control Parameter: time constant of the
sliding surface
P.xi = 0.01;                  % Sliding Control Parameter: importance of beta.
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This controller was simulated with an input path in the shape of a horizontal figure eight with radius 20m and speed 10m/s with the initial vehicle state at the right edge with initial velocity in the inertial Y direction. The vehicle goes around three times. The results from this simulation were very good. Figure 1 shows the vehicles path tracking the figure eight with low error (both variance and bias).

The controller is also capable of working while skidding around fast curves. Figure 2 shows the significant body y velocity as it goes around the circles.

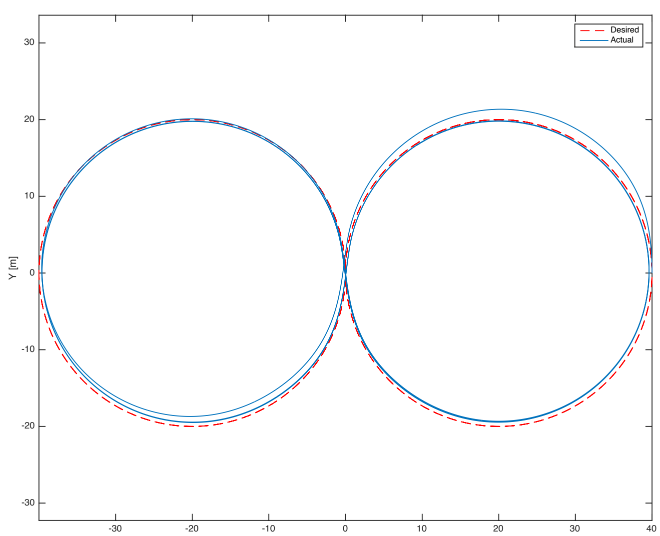

Figure 1. Desired vs. actual path of a vehicle going around a radius 20m figure eight.

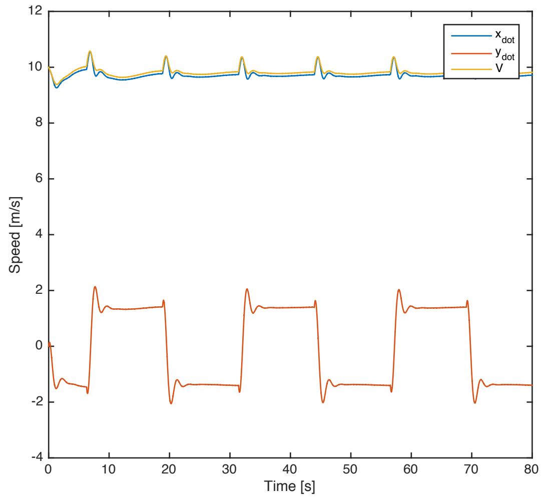

Figure 2. Body fixed velocity and speed of controlled vehicle

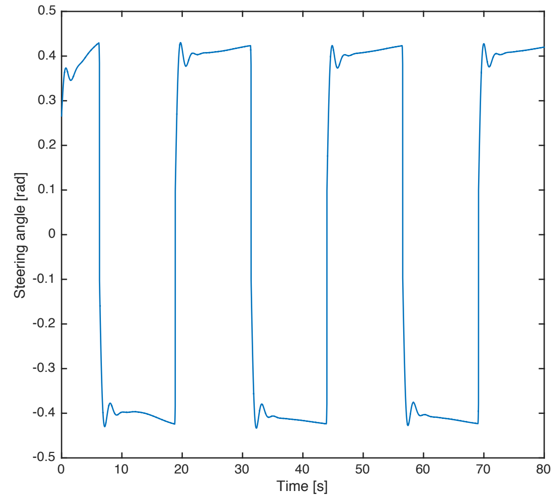

Figure 3. Commanded steering angle

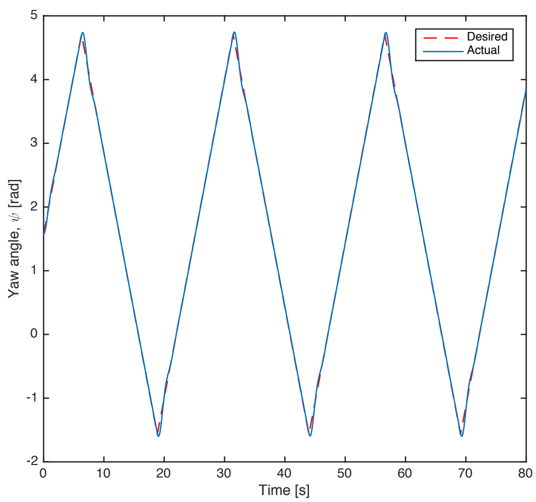

Figure 4. Desired vs. actual yaw angle of vehicle

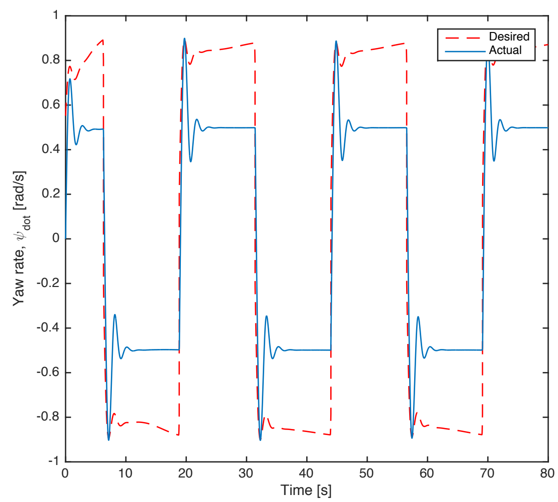

Figure 5. Desired vs. actual yaw rate of vehicle

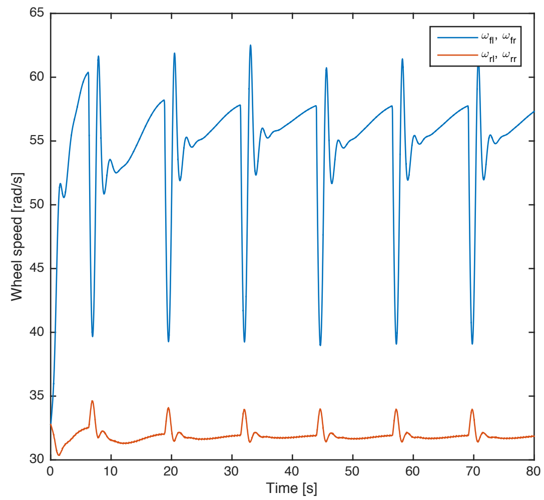

Figure 6. Front and back vehicle wheel speeds

6 Conclusions and Summary

In conclusion, this paper presents a novel path following controller designed to be flexible to the input path and to more fully utilize the capabilities of the vehicle in regards to skidding when necessary. This controller works well in simulation and I hope it motivates further study and even trials on real vehicles in real world scenarios.

7 References

[1] Rajesh Rajamani, Vehicle Dynamics and Control, University of Minnesota, MN; 2006.

[2] Mohammad Ali, Andrew Gray, Yiqi Gao, J. Karl Hedrick, Francesco Borrelli, Multi-Objective Collision Avoidance, In *ASME 2013 Dynamic Systems and Control Conference*, American Society of Mechanical Engineers.